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# The noncommutative effects on the dipole moments of fermions in the standard model

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Abstract: We study the dipole moments, electric dipole moment, weak electric dipole moment, anomalous magnetic moment, anomalous weak magnetic moment, of fermions in the noncommutative extension of the SM. We observe that the noncommutative effects are among the possible candidates to explain the electric and weak electric dipole moment of fermions. Furthermore, the upper bounds for the parameters which carry space-time and space-space noncommutativity can be obtained by using the theoretical and experimental results of the fermion dipole moments.

Keywords: Non-Commutative Geometry, Beyond Standard Model, CP violation.



 

## Contents



## 1. Introduction

The dipole moments of fermions arising from the couplings with the photon and the Z boson provide precise tests of the quantum field theories. In the case of the photon-fermion-fermion ($\gamma f^+ f^-$) coupling there exist two types of dipole moments: the electric dipole moment (EDM) and the anomalous magnetic moment (AMM). On the other hand, the weak dipole moments of the fermions arise from the couplings of $Z f^+ f^-$ vertex and they are two types similar to previous ones: the weak electric dipole moment (WEDM) and the weak anomalous magnetic moment (WAMM). Within the SM the AMM (WAMM) receives its leading value from one-loop corrections, however the EDM (WEDM) can exits at the higher order in the coupling constant since it receives the non-vanishing value through the CP violating effects coming from the CKM matrix elements. Therefore the SM model contribution to EDM and WEDM is irrelevant in the phenomenological analysis and forces one to search the new physics effects, whereas the AMM and WAMM receive the main contribution within the SM.

There are various studies on the the dipole moments of fermions (EDM, WEDM, AMM, WAMM) in the literature [1]–[39]. The EDM of fermions are interesting from the experimental point of view since there are improvements in the experimental limits of charged lepton EDM. EDM of electron, muon and tau have been measured and the present limits are $d_e = (1.8 \pm 1.2 \pm 1.0) \times 10^{-27}$ e-cm [1], $d_\mu = (3.7 \pm 3.4) \times 10^{-19}$ e-cm [2] and $d_\tau = 3.1 \times 10^{-16}$ e-cm [3]. The measurement of the electron EDM has been made using heavy atoms and the first result is obtained as $d_e = -0.2 \pm 3.2 \times 10^{-26}$ e-cm [4]. The search for the EDM of tau using the reaction $e^+e^- \to \tau^+\tau^-$ has been done and the numerical values $\mathrm{Re}[d_\tau] = (1.15 \pm 1.7) \times 10^{-17}$ e-cm, $\mathrm{Im}[d_\tau] = (-0.83 \pm 0.86) \times 10^{-17}$ e-cm were obtained in [5]. In [6], it is emphasized that the dominant contribution to the EDM of lighter leptons comes from the two loop diagrams that involve one power of the Higgs Yukawa couplings. In this work, the CP-violation is assumed to be mediated by neutral Higgs scalars and the EDM of electron is predicted at the order of the magnitude of $10^{-26}$ e-cm. Furthermore, the works in [7]–[9] are related to analysis of the EDM of leptons in supersymmetric models . In the work [10], the lepton EDMs are studied by scaling them with corresponding lepton masses and the EDM of electron is predicted as $10^{-27}$ e-cm. [11] is devoted to the study of the EDM of leptons $e, \mu, \tau$ in the general two Higgs doublet model (2HDM). Lepton EDM in non-degenerate supersymmetric see saw model have been examined in [12] and it was shown that in the minimal supersymmetric seesaw model with non-degenerate heavy neutrino masses, the charged lepton EDMs may be enhanced by several orders of the magnitude compared to the heavy neutrino case. In [13] it has been emphasized that there was another source of EDM inherent in more fundamental models such as supersymmetric models derived from strings and the string models which accommodate the fermion mass hierarchy and mixings generally lead to large EDMs. The EDM of leptons have been studied in the left right supersymmetric models in [14]. [15] is devoted to the sources of fermion dipole moment contributions from R-parity violating or lepton number violating parameters. In [16] the muon electric dipole moment induced by Higgs bosons, third-generation quarks and squarks, charginos and gluinos in the minimal supersymmetric standard model (MSSM) has been studied. The constraints that the non-observation of electric dipole moments imposed on the radiatively-generated CP-violating Higgs sector and on the mechanism of electroweak baryogenesis in the MSSM, were discussed. The tau EDM has been examined in [17] and it was concluded that the stringent and independent bounds on the tau EDM competitive with the high energy measurements, can be established in the low energies experiments.

The naturalness bounds on the dipole moments from new physics has been studied on [18]. The EDMs of quarks are not directly measurable, however they can affect the hadron phenomenology, for example through the escaling violation in deep inelastic lepton-hadron scattering or EDM of nucleons. In this work the upper bounds of the quark EDMs have been estimated as

$$
\begin{aligned}
|d_u^\gamma| &\leq 4.0 \times 10^{-20}\,\text{e-cm}\,, \\
|d_d^\gamma| &\leq 1.5 \times 10^{-19}\,\text{e-cm}\,, \\
|d_s^\gamma| &\leq 3.0 \times 10^{-18}\,\text{e-cm}\,, \\
|d_c^\gamma| &\leq 1.1 \times 10^{-17}\,\text{e-cm}\,, \\
|d_b^\gamma| &\leq 7.0 \times 10^{-17}\,\text{e-cm}\,, \\
|d_t^\gamma| &\leq 1.5 \times 10^{-15}\,\text{e-cm}\,.
\end{aligned}
\tag{1.1}
$$

In [19], the electric dipole form factors for heavy fermions have been calculated in the 2HDM and it was predicted as

$$
\begin{aligned}
|d_\tau^\gamma| &\leq 10^{-23}\,\text{e-cm}\,, \\
|d_t^\gamma| &\leq 10^{-20}\,\text{e-cm}\,, \\
|d_b^\gamma| &\leq 10^{-20}\,\text{e-cm}\,, \\
|d_c^\gamma| &\leq 10^{-21}\,\text{e-cm}\,,
\end{aligned}
\tag{1.2}
$$

where the predictions for model I and II versions of the 2HDM are

$$\begin{aligned}
|d_\tau^\gamma| &\le 10^{-24}\,\text{e-cm}\,,\\
|d_t^\gamma| &\le 10^{-20}\,\text{e-cm}\,,\\
|d_b^\gamma| &\le 10^{-20}\,\text{e-cm}\,,\\
|d_c^\gamma| &\le 10^{-24}\,\text{e-cm}\,.
\end{aligned} \tag{1.3}$$

The EDM of $b$-quark in the general 2HDM and in the 3HDM with $O(2)$ symmetry in the Higgs sector has been calculated in [20] and, even at one loop, the EDM was obtained in the order of $10^{-20}$ e-cm. The work in [21] is devoted to the EDM of top quark in the general 2HDM, including charged Higgs contribution and the numerical value of EDM was obtained at the order of magnitude of $10^{-20}$ e-cm.

The WEDM of an elementary particle is the another signature for the CP violation. An improved test of CP invariance in the reaction $e^+e^- \to \tau^+\tau^-$ on $Z^0$ peak is performed using the data sample recorded between 1991 and 1995 with the OPAL detector at LEP. From the non-observation of CP violation, the upper limits for the real and imaginary part of the WEDM of tau lepton is derived 95% confidence level as $\mathrm{Re}[d_\tau^Z] \le 5.6\times 10^{-18}$ e-cm and $\mathrm{Im}[d_\tau^Z] \le 1.5\times 10^{-17}$ e-cm [22].

In [23] the WEDMs of fermions have been analyzed and it was emphasized their property of being five-dimensional operators in the effective lagrangian. This suggest that WEDM is proportional to $m_f/\Lambda^2$ where $\Lambda$ is the scale of the physics involved. A theoretical work was performed in the framework of the leptoquark models [24] and the WEDM of tau is predicted as $|d_\tau^Z| \sim 10^{-18}$ e-cm. In [25] the WEDM of $\tau$ lepton has been estimated in the order of $10^{-18}$ e-cm by studying the normal and transverse polarization for $\tau^+\tau^-$ pairs produced from unpolarized $e^+e^-$ collisions at the Z-peak.

In [19] the WEDM of heavy fermions have been studied in the framework of the 2HDM and the predicted values read:

$$\begin{aligned}
|d_\tau^Z| &\le 10^{-22}\,\text{e-cm}\,,\\
|d_t^Z| &\le 10^{-19}\,\text{e-cm}\,,\\
|d_b^Z| &\le 10^{-20}\,\text{e-cm}\,,\\
|d_c^Z| &\le 10^{-22}\,\text{e-cm}\,.
\end{aligned} \tag{1.4}$$

These numerical results agree with the predictions of $\tau$ lepton and $b$ quark WEDMs which were obtained in [26] within the MSSM.

The AMM of a fermion receives its leading value from one-loop corrections in the SM and therefore it is not so much sensitive to the new physics effects beyond. However, with the recent anouncement of world average AMM of muon at BNL [27],

$$a_\mu = 11\,659\,203\,(8)\times 10^{-10}\,, \tag{1.5}$$

which has about half of the uncertainty of previous measurements and its SM prediction, a new window is opened for testing the SM and beyond, assuming that the new physics effects can not exceed the deviation. There are various attempts in different models to



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explain the small deviation of the SM result [28]. Furthermore the deviation of AMM of electron (tau) from its SM contribution is $\Delta a_e\,(\Delta a_\tau) \sim 10^{-11}\,(10^{-3})$ [18] and this can be explained by the physics beyond the SM. In [18] AMM of quarks have been also studied and they have been estimated as:

$$
\begin{aligned}
|a_u^\gamma| &\leq 4 \times 10^{-6}\mu_N\,,\\
|a_d^\gamma| &\leq 1.5 \times 10^{-5}\mu_N\,,\\
|a_s^\gamma| &\leq 3 \times 10^{-4}\mu_N\,,\\
|a_c^\gamma| &\leq 1.1 \times 10^{-3}\mu_N\,,\\
|a_b^\gamma| &\leq 7 \times 10^{-3}\mu_N\,,\\
|a_t^\gamma| &\leq 1.4 \times 10^{-1}\mu_N\,,
\end{aligned}
$$

where $\mu_N = \frac{e}{2\,m_{\text{proton}}}$ is the nuclear magneton.

The WAMM of a fermion receives its leading value from one-loop corrections in the SM similar to AMM. Since WAMM is generated through a chirality flip mechanism it is expected to be proportional to the mass of the particle and therefore, the heavy fermions are the candidates for the sizable WAMMs. WAMM of $\tau$ lepton has been studied in the literature extensively [29]–[38]. L3 presented the direct limit on $|\,\mathrm{Re}(a_\tau^Z)| \leq 0.01495$ % CL [29]. Using the data given in 1997 [30] the WAMM has been predicted as $|\,\mathrm{Re}(a_\tau^Z)| \leq$ 0.002795 % CL [31].

This physical quantity has been calculated in the framework of the SM as

$$
a_\tau^Z(SM) = -(2.10 + 0.61i) \times 10^{-6}\,, \tag{1.6}
$$

in [32] and it was studied in [33] in the framework of the 2HDM. The work in [34] is devoted to the calculation of the WAMM in the super symmetric (SUSY) model and it was observed that the WAMM increases with the increasing values of the parameter $tan\,\beta$. The WAMM of $b$-quark has been calculated as

$$
a_b^Z = (3.57 - 1.95i) \times 10^{-4}\,, \tag{1.7}
$$

in [33] and in [35] the numerical values for $c$ and $b$ quark were presented as

$$
\begin{aligned}
a_c^Z &= (-2.80 + 1.09i) \times 10^{-5}\,,\\
a_b^Z &= (2.98 - 1.56i) \times 10^{-4}\,.
\end{aligned} \tag{1.8}
$$

Notice that, similar to the WEDM, WAMM is created by five-dimensional operators in the effective lagrangian and therefore WAMM is proportional to $m_f^2/\Lambda^2$ where $\Lambda$ is the scale of the physics involved [27].

In the present work, we study the noncommutative (NC) effects on the dipole and magnetic moments of fermions in the framework of the SM. The string theory arguments re-motivate the physics on the noncommutative spaces [40, 41] and the quantum field theory over noncommutative spaces [42] has been reached a great interest in recent years .

NC field theory have a non-local structure and the Lorentz symmetry is explicitly violated. These violations have been analyzed in [43, 44]. The renormalizability and the unitarity of NC theories have been studied in the series of works [45, 46] and [47]. The quantum electrodynamics including the noncommutative effects (NCQED) has been examined in [39, 48] and NCQED has been studied in the extra dimensions in [49]. The noncommutativity in non-abelian case has been formulated in [50, 51]. The work in [52] is due to the application of this formulation in to the SM. Noncommutative SM (NCSM) building has been studied in [53] and the determination of triple neutral gauge boson couplings has been done in [54]. In [55] a unique model for strong and electroweak interactions with their unification has been constructed. Furthermore, the phenomenological analysis of the noncommutative effects on some processes has been studied in several works [56]-[61]. In [56], the noncommutative CP violating effects have been examined at low energies and it was ephasized that CP violation due to noncommutative geometry was comparable to the one due to the standard model (SM) only for a noncommutative scale $\Lambda \leq 2\,\mathrm{TeV}$. [57] is devoted to the SM forbidden processes $Z \to \gamma\gamma$ and $Z \to gg$ with the inclusion of the NC effects. In [58] the form factors, appearing in the inclusive $b \to sg$ decay, has been calculated in the NCSM, using the approximate phenomenology and the new operators existing in $b \to sg$ decay due to the NC effects has been obtained in [59]. In the recent work [60], the possible effects of NC geometry on weak CP violation and the untarity triangles have been examined. The work in [61] is devoted to the $Z \to l^+ l^-$ and $W \to \nu_l l^+$ decays, for $l = e, \mu, \tau$, in the SM, including the noncommutative (NC) effects.

In our analysis we observe that the EDM and WEDM of fermions are sensitive to the NC effects and these physical quantities would be informative in the determination of the upper bounds of the NC parameters.

The paper is organized as follows: in section 2, we present the explicit expressions for the EDM, AMM, WEDM and WAMM of fermions in the framework of the NC SM. Section 3 is devoted to discussion and our conclusions.

## 2. The noncommutative effects on the dipole moments of the fermions in the SM

The nature of the space-time changes at very short distances of the order of the Planck length and the non commutativity in the space-time is a possible candidate to describe the physics at these distances. In the noncommutative geometry the space-time coordinates are replaced by hermitean operators $\hat{x}_\mu$ which satisfy the equation [62]

$$[\hat{x}_\mu, \hat{x}_\nu] = i\,\theta_{\mu\nu}\,, \tag{2.1}$$

where $\theta_{\mu\nu}$ is a real and antisymmetric tensor with the dimensions of $[mass^{-2}]$. Here $\theta_{\mu\nu}$ can be treated as a background field relative to which directions in space-time is distinguished and its components are assumed as constants over cosmological scales. Introducing $*$ product of functions, instead of the ordinary one,

$$(f * g)(x) = e^{\frac{i}{2}\,\theta_{\mu\nu}\,\partial^y_\mu\,\partial^z_\nu} f(y)\, g(z)|_{y=z=x}\,. \tag{2.2}$$

it is possible to pass to the noncommutative field theory. The commutation of the hermitean operators $\hat{x}_\mu$ (see eq. (2.1)) holds with this new product, namely,

$$[\hat{x}_\mu, \hat{x}_\nu]_* = i\,\theta_{\mu\nu}\,. \tag{2.3}$$

Since the noncommutative effects are tiny to observe in the low energy physics one would expect that the physical quantities, which do not exist even at the loop levels more than two in the SM, may be sensitive to the NC effects. The EDM and WEDM can exits at the higher order in the coupling constant in the SM since it receives the non-vanishing value through the CP violating effects coming from the CKM matrix elements. Therefore these quantities may be sensitive to the new physics effects coming from the NC extension of the SM. On the other hand, AMM and WAMM of a fermion receive their leading value from one-loop corrections and therefore it is not so much sensitive to the new physics effects beyond the SM. However, with the stringent bounds obtained in the measurements, the deviations of these quantities from their SM values would be a possible candidate to search the NC effects. In the present work we estimated the EDM, WEDM, AMM and WAMM of massive leptons and quarks in the NC extension of the SM.

EDM (WEDM) and AMM (WAMM) exist in the case of the photon-fermion-fermion (Z boson-fermion-fermion) coupling and the effective lagrangian of these quantities are

$$\mathcal{L}_{\mathrm{EDM\,(WEDM)}} = i d^{\gamma(Z)}\,\bar{f}\,\gamma_5\,\sigma_{\mu\nu}\,f\,F^{\mu\nu}\,, \tag{2.4}$$

and

$$\mathcal{L}_{\mathrm{AMM\,(WAMM)}} = a^{\gamma(Z)}\,\frac{e(g)}{4\,m_f}\,\bar{f}\,\sigma_{\mu\nu}\,l\,F^{\mu\nu}\,, \tag{2.5}$$

where $F_{\mu\nu}$ is the electromagnetic field tensor (weak field tensor), $d^{\gamma(Z)}$ is EDM (WEDM) of the fermion $f$ and $a^{\gamma(Z)}$ is AMM (WAMM) of the fermion $f$. Notice that EDM (WEDM) and AMM (WAMM) are proportional to the coefficients $d$ and $a$ in the interactions

$$\begin{aligned} & d\,\hat{n}.\vec{E}\,, \\ & \frac{e(g)}{4\,m_f}\,a\,\hat{n}.\vec{B}\,, \end{aligned} \tag{2.6}$$

respectively, as it can be obtained using the eqs. (2.4) and (2.5).

When the non-commutative effects are switched on there exists a new contribution which is proportional to the a function of the noncommutative parameter $\theta$. The lagrangian for the additional vertex to the $\gamma f^+ f^-$ and $Z f^+ f^-$ interaction reads [52]

$$\begin{aligned} L^f_{A(Z)} = & -\frac{i}{2}\left\{ c^{A(Z)}_{1\,f}\,\bar{f}\left(\frac{1}{2}\theta_{\mu\nu}\gamma_\alpha + \theta_{\nu\alpha}\gamma_\mu\right) L\,\partial^\alpha f + c^{A(Z)}_{2\,f}\,\bar{f}\left(\frac{1}{2}\theta_{\mu\nu}\gamma_\alpha + \theta_{\nu\alpha}\gamma_\mu\right) R\,\partial^\alpha f \right\} \times \\ & \times \left(\partial^\mu V^\nu_{A(Z)} - \partial^\nu V^\mu_{A(Z)}\right), \end{aligned} \tag{2.7}$$

with

$$
\begin{aligned}
c^A_{1\,f} &= \cos\theta_W g' Y^L_f + \frac{1}{2}\, s\, g\, \sin\theta_W\,, \\
c^A_{2\,f} &= \cos\theta_W\, g'\, Y^R_f\,, \\
c^Z_{1\,f} &= -\sin\theta_W\, g'\, Y^L_f + \frac{1}{2}\, s\, g\, \cos\theta_W\,, \\
c^Z_{2\,f} &= -\sin\theta_W\, g'\, Y^R_f\,,
\end{aligned}
\tag{2.8}
$$

where $V^\mu_A$ ($V^\mu_Z$) denotes the photon (Z boson) field, $s = 1\,(-1)$ for $u$ quarks ($d$ quarks and leptons), $Y^L_f = (-1/2, 1/6)$ for ($f$=leptons, $f$=quarks) and $Y^R_f = (-1, 2/3, -1/3)$ for ($f$=leptons, $f$=up quarks, $f$=down quarks).

At this stage we formulate the dipole moments of fermions using their definitions in eqs. (2.6) and the NC extensions of SM given in eq. (2.7). The EDM (WEDM) interaction of fermions are obtained as

$$
L^{f\,NC}_{\mathrm{EDM\,(WEDM)}} = \frac{1}{2}\, e\, c_f\, m_f\, |\Theta_T|\, \hat{p}_i . E^i\,, \tag{2.9}
$$

where we take $\bar{f}\theta_{0i} f = |\Theta_T|\, \hat{p}_i$ and we use $F^{0i} = E^i$. In eq. (2.9) the vector $(\Theta_T)_i$ is responsible for time-space non commutativity and $\hat{p}_i$ is the unit vector in the direction of $(\Theta_T)_i$. Furthermore, $E_i$ is the electric (weak electric) field and $m_f$ is the fermion mass. Finally the EDM (WEDM) of fermions reads

$$
d_f = \frac{1}{2}\, e\, c_f\, m_f\, |\Theta_T|\,, \tag{2.10}
$$

where $c_f = Q_f$ for the EDM case and $c_f = \frac{1}{6\,\sin 2\theta_W}\,(3-8\sin^2\theta_W)$, $-\frac{1}{6\,\sin 2\theta_W}\,(3-4\sin^2\theta_W)$, $-\frac{1}{2\,\sin 2\theta_W}\,(1-4\sin^2\theta_W)$ for $u$ quarks, $d$ quarks and massive leptons respectively, in the case of WEDM.

The AMM (WAMM) can be obtained using the interaction lagrangian

$$
L^{NC}_{f\,\mathrm{AMM\,(WAMM)}} = \frac{1}{2}\, b_f\, m^2_f\, |\Theta_S|\, \hat{\mu} . \vec{B}\,, \tag{2.11}
$$

where we take $\bar{f}\theta_{ij} f = \frac{1}{2}\, \epsilon_{ijk}\, |\Theta_S|\, \hat{\mu}^k$ and we use $\epsilon_{ijk}\, F^{ij} = B_k$. Here $B_k$ is the magnetic (weak magnetic) field . The vector $(\Theta_S)_k$ is responsible for space-space non commutativity and $\hat{\mu}^k$ is the unit vector in the direction of $(\Theta_S)^k$. As a result, the AMM (WAMM) of fermions reads

$$
a_f = \frac{1}{2}\, b_f\, m^2_f\, |\Theta_S|\,, \tag{2.12}
$$

where $b_f = Q_f$ for the AMM case and $b_f = \frac{1}{6\,\sin 2\theta_W}\,(3-8\sin^2\theta_W)$, $-\frac{1}{6\,\sin 2\theta_W}\,(3-4\sin^2\theta_W)$, $-\frac{1}{2\,\sin 2\theta_W}\,(1-4\sin^2\theta_W)$ for $u$ quarks, $d$ quarks and massive leptons respectively, in the case of WAMM. The eqs. (2.10) and (2.12) show that the NC effects on the EDM and WEDM (AMM and WAMM) are proportional to the mass (square of mass) of the fermions and the heavy fermion EDM and WEDM (AMM and WAMM) receive the large contribution from the NC effects. Notice that the AMM and WAMM is due to the intrinsic magnetic moment of the fermion and its is spin independent.



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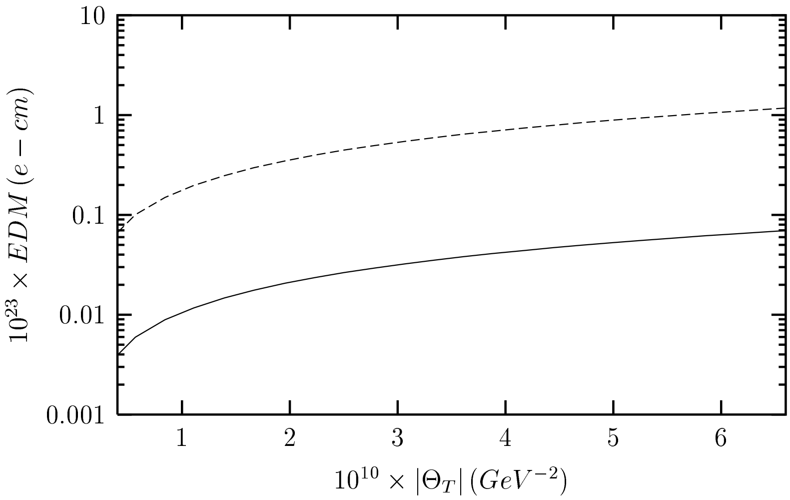


**Figure 1:** The noncommutative parameter $|\Theta_T|$ dependence of lepton EDM. Here the solid (dashed) line represents EDM of $\mu$ ($\tau$) lepton.

## 3. Discussion

This section is devoted to the analysis of the NC effects on the dipole moments of fermions EDM, WEDM, AMM and WAMM, in the framework of the SM. Since the EDM and WEDM can exits at the higher order in the coupling constant in the SM due to the their CP violating nature, they could be sensitive to the new physics effects beyond, NC effects in the present work. The NC effects on the dipole moments deserve to analyze since they can bring comprehensive information in the determination of the bounds of the new NC parameters. On the other hand, AMM and WAMM of a fermion exist in the one-loop order in the SM, and they are not sensitive to the new physics effects beyond, compared to the EDM and WEDM. However, the deviations of the these physical quantities from the SM values can be examined by including the NC effects and the possible constraints can be obtained for the NC parameters.

Our starting point is the experimental result of the electron EDM $|d_e| \leq 3.4 \times 10^{-26}$ e-cm [4]. Here we assume that the source of the EDM is the NC effect in the SM. Using the experimental upper limit of $|d_e|$ and the eq. (2.10) we obtain an upper bound for the NC parameter $|\Theta_T|$ which is responsible for time-space non commutativity, $|\Theta_T| \leq 6.67 \times 10^{-10}(\,\mathrm{GeV}^{-2})$. Now we estimate the upper limits of the EDM of the fermions (leptons $\mu$, $\tau$ and quarks) using the numerical value of $|\Theta_T|$ and the eq. (2.10). Notice that in our predictions we study the absolute values of the dipole moments.

In figure 1, we present the noncommutative parameter $|\Theta_T|$ dependence of lepton EDM. Here the solid (dashed) line represents EDM of $\mu$ ($\tau$) lepton. This figure shows that $\tau$ ($\mu$) lepton EDM can take the numerical values at the order of $10^{-23}$ e-cm ($10^{-24}$ e-cm) in the given interval of the parameter $|\Theta_T|$ . These numerical values satisf the current experimental results, $d_\mu = (3.7 \pm 3.4) \times 10^{-19}$ [2] and $d_\tau \leq 3.1 \times 10^{-16}$ [3].

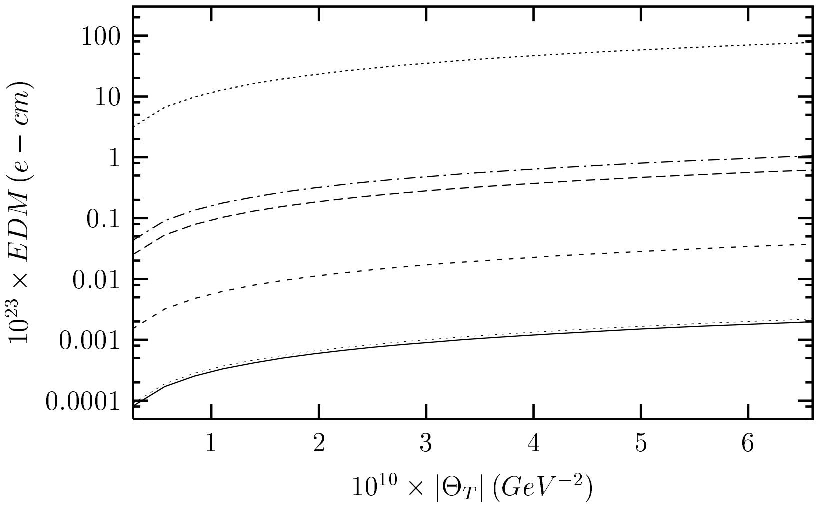


**Figure 2:** The noncommutative parameter $|\Theta_T|$ dependence of quark EDM. Here the solid (dotted, small dashed, dashed, dashed-dotted, dense-dotted) line represents EDM of $u$ ($d$, $s$, $c$, $b$, $t$) quark.

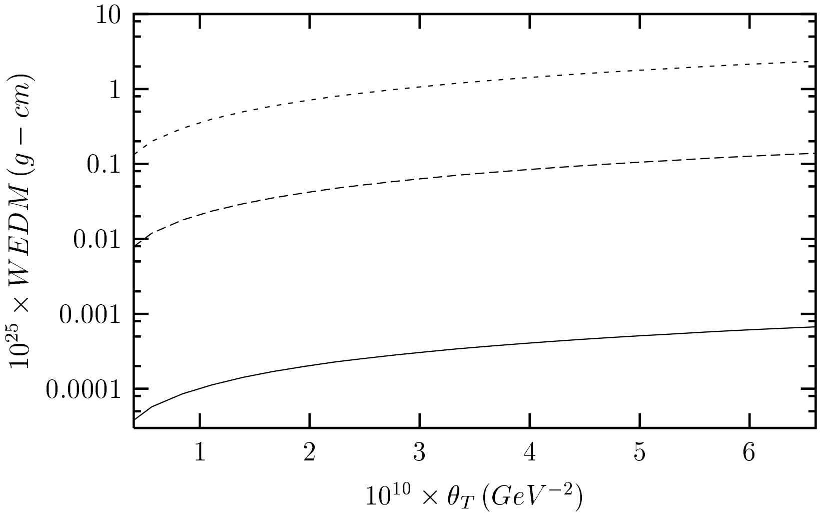


**Figure 3:** The noncommutative parameter $|\Theta_T|$ dependence of lepton WEDM. Here the solid (dashed, small dashed) line represents WEDM of $e$ ($\mu$, $\tau$) lepton.

Figure 2 is devoted to the noncommutative parameter $|\Theta_T|$ dependence of quark EDM. Here the dotted (solid, small dashed, dashed, dashed-dotted, dense-dotted) line represents EDM of $u$ ($d$, $s$, $c$, $b$, $t$) quark. $u$ ($d$, $s$, $c$, $b$, $t$) EDM can receive the numerical values at the order of magnitude of $10^{-26}$ e-cm ($10^{-26}, 5.0 \times 10^{-25}, 10^{-23}, 10^{-23}, 10^{-21}$ e-cm) for $u$ ($d$, $s$, $c$, $b$, $t$) quark. These numerical values are in accordance with the theoretical results presented in eqs. (1.1)–(1.3) and the t-quark EDM which is calculated in [21].

Figure 3, represents the noncommutative parameter $|\Theta_T|$ dependence of lepton WEDM. Here the solid (dashed, small dashed) line represents WEDM of $e$ ($\mu$, $\tau$) lepton. The $\tau$ lepton WEDM can take the numerical values at the order of the magnitude of

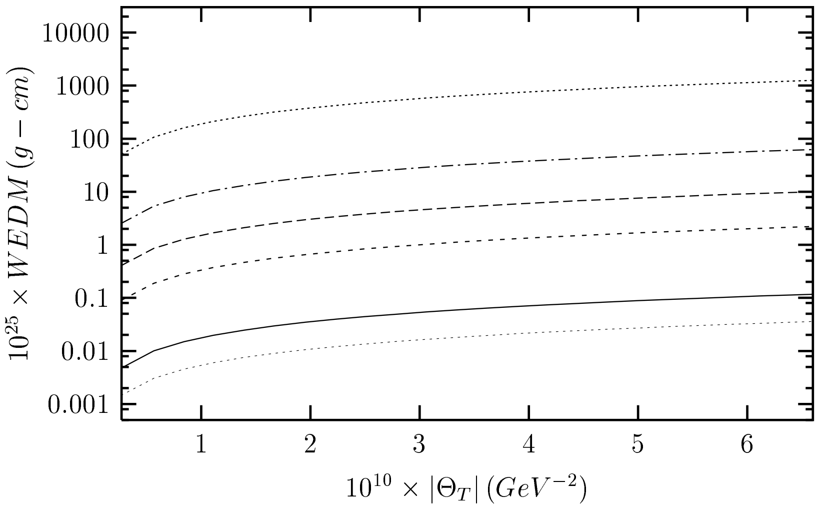


**Figure 4:** The noncommutative parameter $|\Theta_T|$ dependence of quark WEDM. Here the solid (dotted, small dashed, dashed, dashed-dotted, dense-dotted) line represents EDM of $u$ ($d$, $s$, $c$, $b$, $t$) quark.

$10^{-25}\, g-cm$. This results is almost eight order smaller compared to the experimental result $\mathrm{Re}[d_\tau^Z] =\leq 5.6\times 10^{-18}$ e-cm and $\mathrm{Im}[d_\tau^Z] =\leq 1.5\times 10^{-17}$ e-cm [22]. The more accurate measurements in future would make it possible to test NC effects on WEDM more stringently. Furthermore, the theoretical result in the framework of the 2HDM was presented in [19] as $d_\tau^Z \leq 10^{-22}$ e-cm and the $\tau$ lepton WEDM coming from the NC effects is almost three orders smaller than this numerical value, in the given NC parameter region. The electron and $\mu$ lepton WEDM can receive to the numerical values at the order of the magnitude of $10^{-28}$ g-cm and $10^{-26}$ g-cm and they can be tested with more sensitive forthcoming experimental measurements.

In figure 4 we show the noncommutative parameter $|\Theta_T|$ dependence of quark WEDM. Here the solid (dotted, small dashed, dashed, dashed-dotted, dense-dotted) line represents WEDM of $u$ ($d$, $s$, $c$, $b$, $t$) quark. It is observed that the WEDM can take the numerical value at the order of magnitude of $5.0\times 10^{-27}$ e-cm ($10^{-26}, 10^{-25}, 10^{-24}, 10^{-23}, 10^{-22}$ e-cm) for $u$ ($d$, $s$, $c$, $b$, $t$) quark. These upper bounds are smaller compared to the ones obtained in [19] (see eq. 1.4).

Now, we analyze the AMM and WAMM of a fermions in the SM including the NC effects. These physical quantities can exist even in the one-loop level in the SM and therefore they are not so much sensitive to the new physics effects beyond. However, with the accurate analysis of the deviations of the these physical quantities from the SM values it would be possible to test the new effects coming from NC geometry.

The recent anouncement of world average AMM of muon at BNL [27] in eq. (1.5) and the SM prediction forces that there is still a standard deviation from the experimental result and this could possibly be due to the effects of new physics, at present. Now, we respect the assumption that the new physics effects, NC effects in the present work, on the numerical value of muon AMM should not exceed the present experimental uncertainty,

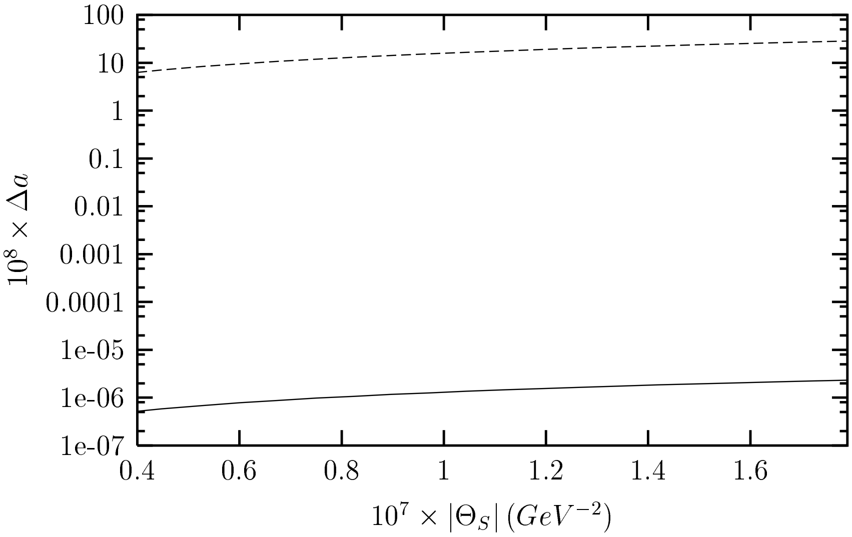


**Figure 5:** The noncommutative parameter $|\Theta_S|$ dependence of lepton $\Delta a$. Here the solid (dashed) line represents $\Delta a$ of $e$ ($\tau$) lepton.

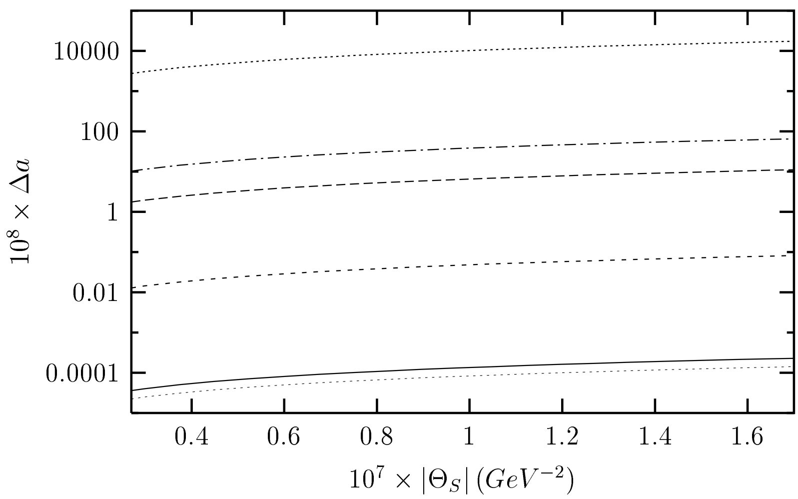


**Figure 6:** The noncommutative parameter $|\Theta_S|$ dependence of quark $\Delta a$. Here the solid (dotted, small dashed, dashed, dashed-dotted, dense-dotted) line represents $\Delta a$ of $u$ ($d$, $s$, $c$, $b$, $t$) quark.

$\sim 10^{-9}$. Using the eq. (2.11), we obtain an upper bound for the NC parameter $|\Theta_S|$ which is responsible for space-space non commutativity, $|\Theta_S| \leq 1.68 \times 10^{-7}(\,\mathrm{GeV}^{-2})$ and using this upper bound, we estimate the NC effects on the AMM of the fermions (leptons $e$, $\tau$ and quarks), which is denoted as $\Delta a$.

In figure 5, we present the noncommutative parameter $|\Theta_S|$ dependence of lepton $\Delta a$. Here the solid (dashed) line represents $\Delta a$ of $e$ ($\tau$) lepton. This figure shows that $\tau$ ($e$) lepton $\Delta a$ can take the numerical values at the order of the magnitude of $5.0 \times 10^{-7}$ ($10^{-14}$). These numerical values are far from the results predicted in [18] $\Delta a_e \sim 10^{-11}\,,(\Delta a_\tau \sim 10^{-3})$.



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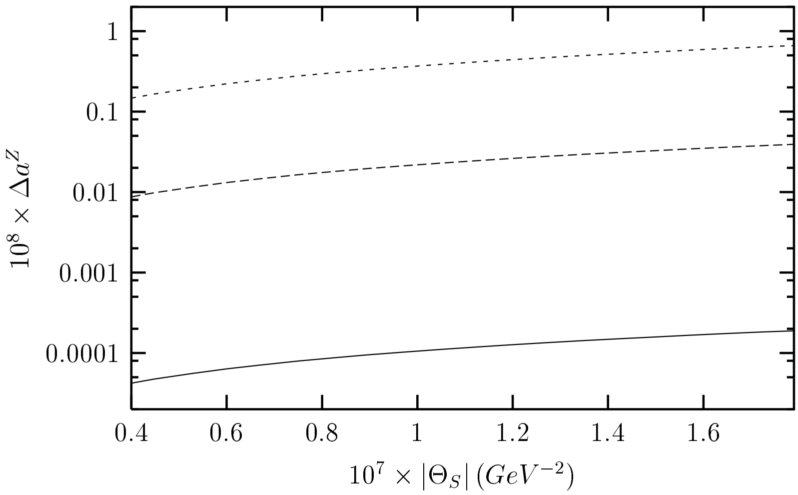


**Figure 7:** The noncommutative parameter $|\Theta_S|$ dependence of lepton WAMM, $\Delta a^Z$. Here the solid (dashed, small dashed) line represents $\Delta a^Z$ of $e$ ($\mu$, $\tau$) lepton.

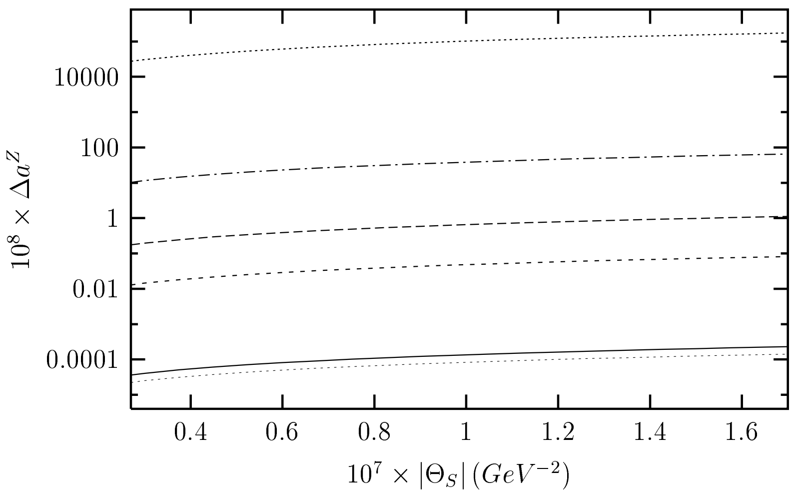


**Figure 8:** The noncommutative parameter $|\Theta_S|$ dependence of quark $\Delta a^Z$. Here the solid (dotted, small dashed, dashed, dashed-dotted, dense-dotted) line represents $\Delta a^Z$ of $u$ ($d$, $s$, $c$, $b$, $t$) quark.

Figure 6 is devoted to the noncommutative parameter $|\Theta_S|$ dependence of quark $\Delta a$. Here the solid (dotted, small dashed, dashed, dashed-dotted, dense-dotted) line represents $\Delta a$ of $u$ ($d$, $s$, $c$, $b$, $t$) quark. $\Delta a$ can receive the numerical values in the order of magnitude of $10^{-12}$ ($10^{-12}, 10^{-9}, 10^{-7}, 5 \times 10^{-7}, 10^{-3}$) for $u$ ($d$, $s$, $c$, $b$, $t$) quark.

Figure 7 represents the noncommutative parameter $|\Theta_S|$ dependence of lepton WAMM, $\Delta a^Z$. Here the solid (dashed, small dashed) line represents $\Delta a^Z$ of $e$ ($\mu$, $\tau$) lepton. The NC effects on WAMM of $e$ ($\mu$, $\tau$)lepton is at the order of the magnitude of $10^{-12}$ ($5.0 \times 10^{-10}$, $10^{-8}$).

Finally figure 8 is devoted to the noncommutative parameter $|\Theta_S|$ dependence of quark $\Delta a^Z$. Here the solid (dotted, small dashed, dashed, dashed-dotted, dense-dotted) line represents $\Delta a^Z$ of $u$ ($d$, $s$, $c$, $b$, $t$) quark. $u$ ($d$, $s$, $c$, $b$, $t$). $\Delta a$ can receive the numerical values in the order of magnitude of $10^{-12}$ ($10^{-12}, 10^{-9}, 10^{-8}, 10^{-6}, 10^{-4}$) for $u$ ($d$, $s$, $c$, $b$, $t$) quark.

At this stage we would like to summarize our results:

- We take the experimental result of the electron EDM $|d_e| \leq 3.4 \times 10^{-26}$ e-cm [4] and obtain an upper bound for the NC parameter $|\Theta_T|$ which is responsible for time-space non commutativity, $|\Theta_T| \leq 6.67 \times 10^{10} (\,\mathrm{GeV}^{-2})$. Using this upper bound, we predict the EDM and WEDM of the fermions existing with the NC effects. We observe that the numerical values of EDM of the leptons $\mu$ and $\tau$ satisfy the current experimental results. The predicted quark EDMs are in accordance with the theoretical results obtained in the literature. Here the $\tau$ lepton WEDM can take the numerical values at the order of magnitude of $10^{-25}$ g-cm. This result is almost eight order smaller compared to the experimental result [22] and almost three orders smaller compared to the theoretical result given by [19]. For the heavy quarks $t$, $b$ and $c$ the predicted values in the NC extension of the SM is not so much far from the ones obtained in the framework of the model I and II versions of the 2HDM [19]. With the more accurate measurements in future, it would be possible to test NC effects on WEDM more stringently.

- We restrict the NC parameter $|\Theta_S|$ which is responsible for space-space non commutativity as $|\Theta_S| \leq 1.68 \times 10^{-7} (\,\mathrm{GeV}^{-2})$ using the assumption that the NC effects on the numerical value of muon AMM should not exceed the present experimental uncertainty, $\sim 10^{-9}$. The numerical values $\Delta a$ for $\tau$ and $e$ lepton we obtain are far from the results predicted in the literature [18].

- The NC effects on the EDM and WEDM (AMM and WAMM) are proportional to the mass (square of mass) of the fermions and the heavy fermion EDM and WEDM (AMM and WAMM) receive the large contribution in the NCSM. Notice that the AMM and WAMM is due to the intrinsic magnetic moment of the fermion and its is spin independent.

Therefore, the more accurate experimental results of the dipole moments of the fermions would be an effective tool to understand the new physics effects due to the NC geometry.

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